АГ 26.11.10

$A(3,2,-1)$, $B(-1,0,2)$

$\beta: 2x-3y+z+7=0$

Нека $n_\alpha(u,v,w)$

$\alpha: u(x-3)+v(y-2)+w(z+1)=0$

$\alpha: u(-1-3)+v(0-2)+w(2+1)=0$

$\alpha: -4u+2v+3w=0$

$\left|\begin{array}{l} -4u+2v+3w=0 \\ 2u-3v+w=0 \end{array}\right.$ (от $P(2;-3,1)$)

$2u=3v-w \Rightarrow -6v+2w-2v+3w=0$

$w=3v-2u$

$4u-2v+9v-6u=-10u+7v$

$v=\frac{10}{7}u \Rightarrow w=\frac{30}{7}u-2u=\frac{16}{7}u$

$\alpha: u(B_1-A_1)+v(B_2-A_2)+w(B_3-A_3)$

$u(x-3)+\frac{10}{7}u(y-2)+\frac{16}{7}u(z+1)=0 \quad /:u$

$x-3+\frac{10}{7}y-\frac{20}{7}+\frac{16}{7}z+\frac{16}{7}=0 \Rightarrow$

$7x+10y+16z-25=0$

$e: \begin{cases} 2x-3y-z=0 \\ 4x+y+5z-28=0 \end{cases}$ | ? y-е на α сбр. e разст $\sqrt{14}$ от O

$\lambda(2x-3y-z)+\mu(4x+y+5z-28)$

$(2\lambda+4\mu)x+(-3\lambda+\mu)y+(-\lambda+5\mu)z-28\mu$

$\sqrt{(2\lambda+4\mu)^2+(-3\lambda+\mu)^2+(-\lambda+5\mu)^2}$

① $\frac{(\lambda_1+\mu_1)x+(\lambda_2+\mu_2)y+(\lambda_3+\mu_3)z}{\sqrt{(\lambda+\mu)^2+(\lambda+\mu)^2+(\lambda_3+\mu_3)^2}}$

Само корена

$$4\lambda^2 + 16\lambda\mu + 16\mu^2 + 9\lambda^2 - 6\lambda\mu + \mu^2 + \lambda^2 - 10\lambda\mu + 25\mu^2$$

$$\sqrt{14\lambda^2 + 42\mu^2}$$

$$\sqrt{14} = \left| \frac{-28\mu}{\sqrt{14(\lambda^2 + 3\mu^2)}} \right|$$

$$\alpha' : 3x - y + 2z + 14 = 0 \quad (\text{при } \lambda = \mu)$$
$$\alpha'' : x + 2y + 3z - 14 = 0 \quad (\text{при } \lambda = -\mu)$$

$A(-1, 1, 2)$ $\qquad e \begin{cases} x - y + 1 = 0 \\ x - z - 2 = 0 \end{cases}$

разст. от A до e

рояване равен $\alpha \perp e$ $\qquad A \in \alpha$

Нека $N_\alpha(u, v, w)$

$$u(x+1) + v(y-1) + w(z-2) = 0$$

$$u - v = 0 \qquad u - w = 0$$

от $\alpha \perp e \Rightarrow \alpha \perp$ на всяка равнина

$$x + y + z - 2 = 0$$

$M = e \cap \alpha$

$$\left| \begin{array}{l} x - y + 1 = 0 \\ x - z - 2 = 0 \\ x + y + z - 2 = 0 \end{array} \right\} + \Rightarrow \qquad x = 1, \; y = 2, \; z = -1$$

$$M(1, 2, -1) \quad | \quad \overrightarrow{AM}(2, 1, -3)$$

$$|\overrightarrow{AM}| = \sqrt{4 + 1 + 9} = \sqrt{14}$$

$A(2,-1,1)$ $\quad g: \begin{cases} x+2y-z+5=0 \\ x-y+2z-1=0 \end{cases}$

$\alpha: 3x-y+2z+10=0$

? $e=?$ $\quad e$ минава през $A \Rightarrow$

$e \parallel \alpha \quad e=\alpha$ (лежат в 1 равнина)

$\beta \in \mathcal{U}_g$

заместване с (A) $\beta: \lambda(x+2y-z+5)+\mu(x-y+2z-1)$

$\lambda(2-2-1+5)+\mu(2+1+2-1)$

$4\lambda+4\mu=0 \Rightarrow \lambda=-\mu$

$\beta: y-x+2=0$

~~γ~~ $\gamma: 3x-y+2z+d=0$ (от α)

от $A \Rightarrow 6+1+2+d=0 \Rightarrow d=-9$

$\gamma: 3x-y+2z-9=0$

$e=\beta \times \gamma$

$M(2,7,1)$ $\quad \alpha: x-4y+z+7=0$

① ? M' ост. симетр на M относно α

② Директорните косинуси на посоката от M към M'

построяваме $h \perp \alpha \quad M \in h$

$h: \begin{cases} x=2+s \\ y=7-4s \\ z=1+s \end{cases}$ (s от α $(1,-4,1)$ и $M(2,7,1)$)

$A=h \times \alpha$

$\alpha: 2+s-4(7-4s)+1+s+7=0$

③

$18s - 18 \Rightarrow s = 1$

от $h\,(2+s, 7-4s, 1+s) \Rightarrow A(3,3,2)$

Нека $M'(x'y'z')$ тогава

$$\left.\begin{array}{l} 3 = \frac{2+x'}{2} \\ 3 = \frac{7+y'}{2} \\ 2 = \frac{1+z'}{2} \end{array}\right\} \Rightarrow \begin{array}{l} x' = 4 \\ y' = -1 \\ z' = 3 \end{array}$$

② Директиони косинуси

$\overrightarrow{MM'}\ (2;-8,2)$

$|\overrightarrow{MM'}| = \sqrt{4+64+4} = \sqrt{72} = 6\sqrt{2}$

$\frac{\overrightarrow{MM'}}{|\overrightarrow{MM'}|} = \left(\frac{1}{3\sqrt{2}}, \frac{-4}{3\sqrt{2}}, \frac{1}{3\sqrt{2}}\right)$ $(\text{от } \overrightarrow{MM'}(2-8-2))$